Date: June 14, 1992

To: O. Alifanov, E. Artyukhin and Y. Jarny

From: J.V. Beck

Subject: COMMENTS ON THE ITERATIVE, CONJUGATE GRADIENT REGULARIZATION METHODS FOR THE IHCP AND PE

Let me first say that there are many excellent features of the iterative, conjugate gradient regularization method for solving the IHCP and PE problems. I expect to take advantage of it in the future. However, there are number of features that I find to be unsatisfactory. Let me tell you about them.

The case that I wish to consider is one with measurement errors and a finite number of measurements. The errors are to be real, having correlated and biased aspects.

1. In the IHCP, I believe that it is misleading to give values of the heat flux at the end of the fixed time domain. These values are not estimated with the same confidence as the others and can be quite unreliable. For example, suppose that the heat flux suddenly changes a dimensionless time of 0.04 before the designated time interval. The values of the heat flux can not be accurately predicted at the end portion. To suggest that it can does not seem honest to me.

2. The answers of the heat flux or other parameters depends upon the initial estimates of the unknowns. If q is set equal to zero at the beginning, the estimates will be slightly different than those for q = 1.

3. The method does not encourage "parsimony" in the selection of parameters. The method will give an answer even if there is extremely high correlation among the parameters. Will it even work for the case of no unique minimum of the sum of squares function? The engineer should be alerted if the problem does not have a distinct minimum. The method seems to encourage the habit (a bad one in my opinion) of choosing too many components, particularly in the thermal conductivity function.

4. The method has not encouraged the development of the determination of confidence intervals. It has not encouraged the examination of the residuals. It has not encouraged the use of statistics. I seem to find in the iterative regularization method that the residuals are somewhat variable one iteration to

the next. This does not encourage to examination of the residuals, if true.

5. The choice of the final time in the IHCP I find to be unsatisfactory. The answers at the beginning should not depend upon the distant future. There does seem to be dependence in the numbers that I obtained.

6. The method requires more iterations as the number of components is increased. This is unavoidable, isn't it? For two and three dimensional problems the amount of computation increases dramatically. I would think that the method should be sequential in time to reduce some of these iterations.

7. The method has not encouraged engineering insight for simple cases or so it seems to me.

8. The method does not naturally require the computation of sensitivity coefficients. (I admit that for large problems that they might be expensive to compute.) The sensitivity coefficients are useful for

a. design of experiments and insight
b. calculation of the confidence intervals
c. discrimination between competing mathematical models

9. The method is much harder to understand and teach that the use of sensitivity coefficients.

10. For each new case, another adjoint equation must be developed and the development is not easy.

After saying the above, I believe that the method is very general and powerful. In general, it may well be more general and more powerful than many other methods. Furthermore,I expect to use certain features of it in the future. However, there are certain weaknesses as I see it, some of which are not shared by the simpler methods of function specification and the use of sensitivity coefficients. These methods certainly have their weaknesses also but they have certain appeal from engineering insight and adaptability perspectives that may not be present in the iterative regularization method.

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